# Students' Strategies for Addition and Subtraction Within 20

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In this study we individually interviewed 127 students in Years 3 and 4 to investigate their strategies for solving addition and subtraction problems within 20. The problems that students were asked to solve were carefully planned to represent particular problem sets, and strategy use was coded in detail to delineate retrieval from different decomposition strategies. The findings highlight the prevalent use of decomposition strategies for single-digit problems that sum to between 10 and 20 and the use of counting strategies for corresponding subtraction problems.

The expectation that by a particular year level, students are able to recall answers to addition and subtraction problems (i.e., know answers from memory) is often made explicit in mathematics curricula. 'Recall', commonly termed retrieval in the research literature, refers to the direct retrieval of an answer from a store of facts held in long term memory (Ashcraft, 1995). Back-up procedures (encompassing any procedure other than retrieval) include Decomposition Strategies, which involve partitioning and regrouping operands to make use of known facts (e.g., the bridging-10 strategy and the near-doubles strategy) and Counting Strategies (e.g., the count-on from-larger strategy, also called the min-counting strategy). There is strong evidence in the cognitive sciences that retrieving an answer from memory represents a qualitatively different action than computing an answer using a highly automated back-up procedure (e.g., Polspoel et al., 2017) and that retrieval develops as problem-answer associations strengthen in memory through the correct application of back-up procedures (Siegler, 1996).

While the importance of students learning to retrieve certain facts for solving addition and subtraction problems is clear, there is surprisingly little research to illuminate exactly what problems they do come to solve using retrieval (i.e., just know) and what problems they continue to solve using Decomposition Strategies. The aim of the current study was to investigate the strategies students use in Years 3 and 4 to solve addition and subtraction problems within 20. (In this paper we refer to specific procedures as strategies, capitalise the name of strategy types, and use lower case when referring to named strategies.)

## Curriculum Context

In Australia, the written mathematics curriculum sets out what students are expected to be able to demonstrate by the end of each year level in the achievement standards. In 2022, changes to the mathematics curriculum were made to reflect "a stronger focus on students mastering the essential mathematical facts, skills, concepts and processes, and being introduced to these at the right time" and include "lifting standards for mathematics in Year 1 in relation to addition and subtraction" (Australian Curriculum Assessment and Reporting Authority [ACARA], n.d.). By the end of Year 2, students are now expected to recall and demonstrate proficiency with addition and subtraction facts within 20 (ACARA, version 9, n.d.); whereas previously, students were expected to recall addition facts for single-digit numbers by the end of Year 3 (ACARA, version 8.4, n.d.). The change in year level brings this particular standard to be more in line with curricula in other countries. For example, in the US, students are expected to "know from memory all sums of two one-digit numbers" by the end of second grade (Council of Chief State School Officers [CCSSO], 2010). In

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England, Year 2 students are to be taught to recall and fluently use addition and subtraction facts to 20 (Department of Education, 2013).

Expanding the Australian curriculum standard to encompass (i) the operation of subtraction as well as addition, and (ii) sums within 20 rather than sums produced by single-digit operands, represents a marked increase in the number of problems explicitly referred to in the standard (as illustrated in Figure 1). Excluding problems with an operand of 0 and 1, and limiting problems to have two operands only, there are 21 single-digit addition problems with sums below 10 (Problem Set 1), 7 single-digit addition problems with sums equal to 10 (Problem Set 2), and 36 single-digit addition problems with sums between 10 and 20 (Problem Set 3). Changing the wording of the standard to include sums within 20 adds another 88 single-digit/double-digit problems (Problem Set 4). Including corresponding subtraction problems further increases the number of problems two-fold. Thus, the change in curriculum wording translates to a substantial increase in the number of problems explicitly referred to in the standard, from 64 problems to 304 problems.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2+2=4 | 4-2=2 | | | | | | KEY: | Set 1 | sum < 10 | | |
| 2+3=5 | 5-3=2 | | | | | | | | | | |
| 3+2=5 | 5-2=3 | 3+3=6 | 6-3=4 | | | | | Set 2 | sum = 10 | | |
| 2+4=6 | 6-4=2 | 3+4=7 | 7-4=3 | | | | | | | | |
| 4+2=6 | 6-2=4 | 4+3=7 | 7-3=4 | 4+4=8 | 8-4=4 | | | Set 3 | 10 < sum < 20 both single digit | | |
| 2+5=7 | 7-5=2 | 3+5=8 | 8-5=3 | 4+5=9 | 9-5=4 | | | | | | |
| 5+2=7 | 7-2=5 | 5+3=8 | 8-3=5 | 5+4=9 | 9-4=5 | 5+5=10 | 10-5=5 | Set 4 | 10 < sum ≤ 20 one single digit | | |
| 2+6=8 | 8-6=2 | 3+6=9 | 9-6=3 | 4+6=10 | 10-6=4 | 5+6=11 | 11-6=5 | | | | |
| 6+2=8 | 8-2=6 | 6+3=9 | 9-3=6 | 6+4=10 | 10-4=6 | 6+5=11 | 11-5=6 | 6+6=12 | 12-6=6 | | |
| 2+7=9 | 9-7=2 | 3+7=10 | 10-7=3 | 4+7=11 | 11-7=4 | 5+7=12 | 12-7=5 | 6+7=13 | 13-7=6 | | |
| 7+2=9 | 9-2=7 | 7+3=10 | 10-3=7 | 7+4=11 | 11-4=7 | 7+5=12 | 12-5=7 | 7+6=13 | 13-6=7 | 7+7=14 | 14-7=7 |
| 2+8=10 | 10-8=2 | 3+8=11 | 11-8=3 | 4+8=12 | 12-8=4 | 5+8=13 | 13-8=5 | 6+8=14 | 14-8=6 | 7+8=15 | 15-8=7 |
| 8+2=10 | 10-2=8 | 8+3=11 | 11-3=8 | 8+4=12 | 12-4=8 | 8+5=13 | 13-5=8 | 8+6=14 | 14-6=8 | 8+7=15 | 15-7=8 |
| 2+9=11 | 11-9=2 | 3+9=12 | 12-9=3 | 4+9=13 | 13-9=4 | 5+9=14 | 14-9=5 | 6+9=15 | 15-9=6 | 7+9=16 | 16-9=7 |
| 9+2=11 | 11-2=9 | 9+3=12 | 12-3=9 | 9+4=13 | 13-4=9 | 9+5=14 | 14-5=9 | 9+6=15 | 15-6=9 | 9+7=16 | 16-7=9 |
| 2+10=12 | 12-10=2 | 3+10=13 | 13-10=3 | 4+10=14 | 14-10=4 | 5+10=15 | 15-10=5 | 6+10=16 | 16-10=6 | 7+10=17 | 17-10=7 |
| 10+2=12 | 12-2=10 | 10+3=13 | 13-3=10 | 10+4=14 | 14-4=10 | 10+5=15 | 15-5=10 | 10+6=16 | 16-6=10 | 10+7=17 | 17-7=10 |
| 2+11=13 | 13-11=2 | 3+11=14 | 14-11=3 | 4+11=15 | 15-11=4 | 5+11=16 | 16-11=5 | 6+11=17 | 17-11=6 | 7+11=18 | 18-11=7 |
| 11+2=13 | 13-2=11 | 11+3=14 | 14-3=11 | 11+4=15 | 15-4=11 | 11+5=16 | 16-5=11 | 11+6=17 | 17-6=11 | 11+7=18 | 18-7=11 |
| 2+12=14 | 14-12=2 | 3+12=15 | 15-12=3 | 4+12=16 | 16-12=4 | 5+12=17 | 17-12=5 | 6+12=18 | 18-12=6 | 7+12=19 | 19-12=7 |
| 12+2=14 | 14-2=12 | 12+3=15 | 15-3=12 | 12+4=16 | 16-4=12 | 12+5=17 | 17-5=12 | 12+6=18 | 18-6=12 | 12+7=19 | 19-7=12 |
| 2+13=15 | 15-13=2 | 3+13=16 | 16-13=3 | 4+13=17 | 17-13=4 | 5+13=18 | 18-13=5 | 6+13=19 | 19-13=6 | 7+13=20 | 20-13=7 |
| 13+2=15 | 15-2=13 | 13+3=16 | 16-3=13 | 13+4=17 | 17-4=13 | 13+5=18 | 18-5=13 | 13+6=19 | 19-6=13 | 13+7=20 | 20-7=13 |
| 2+14=16 | 16-14=2 | 3+14=17 | 17-14=3 | 4+14=18 | 18-14=4 | 5+14=19 | 19-14=5 | 6+14=20 | 20-14=6 | | |
| 14+2=16 | 16-2=14 | 14+3=17 | 17-3=14 | 14+4=18 | 18-4=14 | 14+5=19 | 19-5=14 | 14+6=20 | 20-6=14 | | |
| 2+15=17 | 17-15=2 | 3+15=18 | 18-15=3 | 4+15=19 | 19-15=4 | 5+15=20 | 20-15=5 | | | | |
| 15+2=17 | 17-2=15 | 15+3=18 | 18-3=15 | 15+4=19 | 19-4=15 | 15+5=20 | 20-5=15 | | | | |
| 2+16=18 | 1816=2 | 3+16=19 | 19-16=3 | 4+16=20 | 20-16=4 | | | 8+8=16 | 16-8=8 | | |
| 16+2=18 | 18-2=16 | 16+3=19 | 19-3=16 | 16+4=20 | 20-4=16 | | | 8+9=17 | 17-9=8 | | |
| 2+17=19 | 19-17=2 | 3+17=20 | 20-17=3 | | | | | 9+8=17 | 17-8=9 | 9+9=18 | 18-9=9 |
| 17+2=19 | 19-2=17 | 17+3=20 | 20-3=17 | | | | | 8+10=18 | 18-10=8 | 9+10=19 | 19-10=9 |
| 2+18=20 | 20-18=2 | | | | | | | 10+8=18 | 18-10=8 | 10+9=19 | 19-9=10 |
| 18+2=20 | 20-2=18 | | | | | | | 8+11=19 | 19-11=8 | 9+11=20 | 20-11=9 |
| | | | | | | | | 11+8=19 | 19-8=11 | 11+9=20 | 20-9=11 |
| | | | | | | | | 8+12=20 | 20-12=8 | 10+10=20 | 20-10=10 |
| | | | | | | | | 12+8=20 | 20-8=20 | | |

Figure 1. Addition problems with two operands that have sums within 20 and corresponding subtraction problems (excluding problems with operands of zero and one) divided into problem sets.

While the new wording of the Australian standard has increased the number of problems explicitly mentioned in the standard, it does not specify precisely which of these problems students are expected to solve using retrieval (i.e., just know) by the end of Year 2. The US standard makes it clear that by the end of second grade, students are expected to retrieve answers to addition problems in Problems Set 1, 2 and 3, but is silent about Problem Set 4 and the strategies that students are expected to use to solve corresponding subtraction problems. The UK standard suggests that students are taught to use a combination of retrieval and Decomposition Strategies for solving both addition and subtraction problems in Problem Sets 1 – 4, but does not specify any particular facts that should be retrieved by this stage. The Australian standard is similar to the UK standard but is even less precise in that it suggests that students are expected to use retrieval and other 'proficient' strategies—meaning, possibly, fast and accurate Decomposition Strategies but also Counting Strategies (if the addend counted on, or the subtrahend counted down, is small). It seems straightforward to surmise that students should learn to retrieve answers to addition problems that add to 10, and to problems that involve a doubles fact, given these facts are commonly used in Decomposition Strategies. But beyond these particular problems, what other facts should students come to 'just know'?

## Literature Review

The importance of students learning to retrieve certain facts is clear in the literature; what is not clear is exactly what facts they should be able to retrieve. As a corollary, it is not clear if solving some problems within 20 using Decomposition Strategies is just as 'good' (i.e., acceptable in terms of standards) as solving them using retrieval, if these strategies are executed with speed and accuracy. There are at least three reasons why this is so.

First, students' retrieval use has not always been distinguished from their use of Decomposition Strategies. Retrieval and Decomposition Strategies have often been combined in studies and called *Retrieval-Based Strategies* (Canobi, 2009) *or Memory-Based Strategies*. For example, Geary (2011) examined students' use of Memory-Based Strategies to solve simple addition problems (problems with single-digit operands that sum to 10 or less) and complex addition problems (problems with one single-digit operand and one double-digit operand like 17 + 6). Geary found US students at the beginning of Grade 1 used Memory-Based Strategies to solve 19% of simple addition problems and 7% complex addition problems. The main finding was that students' use of Memory-Based Strategies (i.e., the number of problems solved using these strategies) predicted their mathematics achievement in Grade 1, as well as their growth in mathematics achievement assessed at the end of fifth grade - after controlling for domain-general factors (e.g., IQ, processing speed and working memory capacity). What is not clear from this study is what facts were retrieved and what facts were derived using Decomposition Strategies. Similarly, in our own research, we provided evidence indicating the importance of students using Retrieval-Based Strategies to solve single-digit addition problems by Year 3, but have not expanded on which facts they should be able to retrieve (Hopkins et al., 2022).

Second, when students' use of retrieval has been recorded separately, their use of different back-up strategies has not been delineated. For example, Barrouillet et al. (2008) examined the subtraction strategies used by Grade 3 French students. They coded students' strategies as (i) retrieval, (ii) applying the corresponding known addition fact, or (iii) using an algorithmic procedure. Findings indicated that students used retrieval on 19% of problems, applied the corresponding known addition fact on 28% of problems and applied an algorithmic procedure on 53% of problems. Barrouillet et al. described algorithmic procedures as being Counting Strategies, but the methods they described did not allow them to discern students' use of Decomposition Strategies.

Third, very few studies have considered students' use of addition *and* subtraction strategies in the same study (i.e., with the same participants). This is a noteworthy omission since it restricts the depth of investigation possible. For example, if students do not solve subtraction problems using corresponding addition facts (i.e., using their knowledge of fact families), it could be because they do not know these addition facts or because they do not apply these facts by making use of the *complementary relationship* between addition and subtraction (Canobi, 2009). If strategies for both operations are investigated together, then the first conjecture could be researched. Furthermore, in studies that have concentrated on one operation and not the other, problem groups across the operations have been treated differently so that synthesising findings becomes a complicated endeavour. For example, Geary (2011) referred to simple addition and complex addition where problems were akin to those included in Problem Sets 1 and 2, and Problem Set 3 respectively (shown in Figure 1). Like many researchers studying students' addition strategies, Geary did not include problems from Problem Set 4. In other studies that have focused on students' subtraction strategies, problems in Problem Set 4 have been included (e.g., Robinson, 2001), along with problems in other sets, but these have been grouped differently to how addition strategies have been grouped. For example, Barrouillet et al. (2008) divided problems into three sets that included subtraction problems with, (i) a minuend less than 10, (ii) a minuend greater than 10, and (iii) a minuend of 10. These sets do not correspond to how addition problems sets are generally constructed. Furthermore, it means that problems like 16 – 12 = 4 are categorised differently to a problem like 16 – 4 = 12; yet these two problems are related to the same fact family.

## The Current Study

The aim of the current study was to investigate the strategies students use in Years 3 and 4 to solve addition and subtraction problems within 20. To our knowledge, this is the first study where students' addition *and* subtraction strategies for solving problems that represent *all* problem sets shown in Figure 1 were investigated, and where strategy use was recorded in detail to distinguish between the different types of Decomposition Strategies (e.g., bridging-10 strategy and a near-doubles strategy). The research questions addressed were:

- How often do students use each strategy type to solve addition and subtraction problems within 20?
- How efficient (in terms of accuracy and speed) are students' use of particular back-up strategies?

Due to space limitations, initial findings addressing only the first research question are presented here. The study is framed by a social cognitive perspective of students' strategy development, which posits that individual factors (e.g., experience), as well as actions of significant others (e.g., teachers), and environmental factors (e.g., intended curriculum), influence the strategies students utilise to solve mathematical problems. Thus, investigating the strategies students use provides insights into factors that have influenced their learning.

## Methods

Participants included 127 Year 3 or 4 students from four independent schools in metropolitan Melbourne, ranging from average socioeconomic status (SES) (47th percentile; one school) to high SES (82-95th percentile; three schools). National assessments of Numeracy in Year 3 showed all schools to be achieving as expected for SES. Year 3 was selected given that the standard relating to retrieval targeted students in this year up until most recently. Year 4 was also included as students were organised into composite Year 3/4. Data were collected toward the end of the year. Given there were too many problems to ask each student to solve in one interview (around 20 minutes long), systematic sampling was used to select the problem set each participant would solve: 43 students solved all the problems in Problem Set 1, 43 students solved the same selection of 44 problems in Problem Set 3, and 41 solved the same selection of 44 problems in Problem Set 4. Selected problems represented the range of problems in the set and included fact families (e.g., 6 + 8 = 14, 8 + 6 = 14, 14 – 8 = 6 and 14 – 6 = 8). Participants also solved all 14 problems in Problem Set 2. Thus, in total participants solved 57 or 58 problems.

Students were individually withdrawn from their classroom to work with the research assistant (RA) in a quiet room nearby. Problems were presented on a computer screen one at a time, but in a random order: addition problems were presented first, then, after a short break, subtraction problems were presented. After solving each problem, the student called out their answer and the RA immediately pressed the space bar thereby removing the problem and stopping a timer, which recorded the time between problem presentation and answer. After each problem, the student was asked to explain the strategy they had used. The strategy was coded in situ by the RA if the self-report was consistent with what she had observed. If it wasn't (which was rare), she prompted the student further and coded what the student then reported. Tests of validity were conducted using reaction time analysis. The approach taken here to identify strategy use on a problem-by-problem basis, utilising self-report combined with observation, has been shown to provide valid data representing students' solution strategies for addition (Siegler, 1987) and subtraction (Robinson, 2001).

For addition problems within 20, 14 strategies were coded and labelled, and categorised into four main strategy types (see Table 1 for details). Given previous studies have generally not included addition problems from Problem Set 4, new codes/labels were needed to indicate that an operand had been split (i.e., partitioned into standard place value units) and the unit digits considered first. (See examples of the *split-retrieval, split-counting*, and *split-decomposition* strategies in Table 1). We also used a separate code for the decomposition strategy that made use of the known fact 5 + 5 and labelled this the *fives strategy.* (Since 5 + 5 = 10 can be considered a doubles fact or an add-to-ten fact, its use in a decomposition strategy could be labelled a near-doubles strategy or a bridging-10 strategy. Thus, depending on how the fact is represented, the frequency of one strategy over the other strategy would be inflated, and so it was coded separately.)

**Table 1**

*Addition Within 20: Strategy Types and Strategies*

| Strategy type | Examples | Self-report | Strategies |
|---|---|---|---|
| Retrieval | 3 + 2 = 5 | Just knew it | |
| Decomposition | 13 + 6 = 19 | (3 + 6 knew it) + 10 | Split-retrieval |
| | 8 + 5 = 12 | (8 + 2 knew it) + 3 | Bridging-10 |
| | 6 + 7 = 14 | (7 + 7 knew it) – 1 | Near-doubles (overshoot) |
| | 5 + 8 = 13 | (5 + 5 knew it) + 3 | Fives |
| | 4 + 9 = 13 | (4 + 10 knew it) – 1 | Add-10 (overshoot) |
| | 13 + 4 = 17 | (3 + 3 knew it + 1) + 10 | Split-decomposition |
| Counting | 5 + 9 = 14 | 9; 10,11,12,13,14 | Count-on-from larger |
| | 13 + 6 = 19 | (3; 4, 5, 6, 7, 8, 9) + 10 | Split-count |
| Other | 8 + 9 = 17 | (8 – 1) + (9 + 1) = 7 + 10 = 17 | Equalise |

*Note.* Other Strategies included multiplication, skip-counting, counted all, and counted-on-from-smaller strategies; '*don't know*' was also coded as well as strategies not-elsewhere recorded.

For subtraction problems within 20, 14 strategies were coded and labelled, and categorised into the same four strategy types. When coding the Decomposition Strategies students used for subtraction, we were interested to know whether students applied these strategies to take away the subtrahend (labelled 'down') or if they used indirect addition and started at the subtrahend (labelled 'up'). Solving subtraction problems using indirect addition represents an application of the complementary relationship between addition and subtraction, and is a very efficient way to solve multi-digit subtraction problems (e.g., 75 – 59 = 1 + 15) (Van Der Auwera et al., 2022). To date, subtraction by indirect addition has not been thoroughly investigated for problems within 20.

**Table 2**

*Subtraction Within 20: Strategy Types and Strategies*

| Strategy type | Examples | Self-report | Strategies |
|---|---|---|---|
| Retrieval | 10 – 6 = 4 | Just knew it | |
| Known addition | 10 – 6 = 4 | Knew 6 + 4 = 10 | |
| Decomposition | 14 – 3 = 11 | (4 – 3 knew it) + 10 | Split-retrieval |
| | 15 – 7 = 8 | (15 – 5 knew it) – 2 | Bridging-10 (down) |
| | 12 – 7 = 8 | (12 – 6 knew it) – 1 | Near-doubles (down) |
| | 15 – 7 = 8 | $7 + \boxed{3 + 5}$ | Bridging-10 (up) |
| | 17 – 8 = 9 | $8 + \boxed{8 + 1}$ | Near-doubles (up) |
| | 12 – 8 = 4 | (12 – 10 knew it) + 2 | Subtract-10 (overshoot) |
| | 15 – 11 = 4 | (15 – 10 knew it) – 1 | Subtract-10 |
| Counting | 10 – 6 = 4 | 10; 9, 8, 7, 6, 5, 4 | Count-by-ones (down) |
| | 10 – 6 = 4 | 6; 7, 8, 9, 10 | Count-by-ones (up) |
| | 18 – 3 = 12 | 10 + (8; 7, 6, 5) | Split-count |
| Other | 10 – 6 = 4 | 10 fingers-dropped 6 and counted or looked to see 4 remaining | Model-all |
| | 11 – 6 = 5 | (11 – 1) – (6 – 1) = 10 – 5 = 5 | Equalise |

*Note.* Other codes included don't know and not-elsewhere recorded.

## Initial Results and Discussion

The frequency of students' strategies for addition and subtraction problems within 20 are shown in Figures 2 and 3 respectively. Percentages presented in each figure are based on all trials (correct and incorrect), where a trial represents each time a problem was solved. These encompassed 903 trials for Problem Set 1 (43 students x 21 problems), 889 trials for Problem Set 2 (127 students x 7 problems), 946 trials for Problem Set 3 (43 students x 22 problems), and 902 trials for Problem Set 1 (41 students x 22 problems).

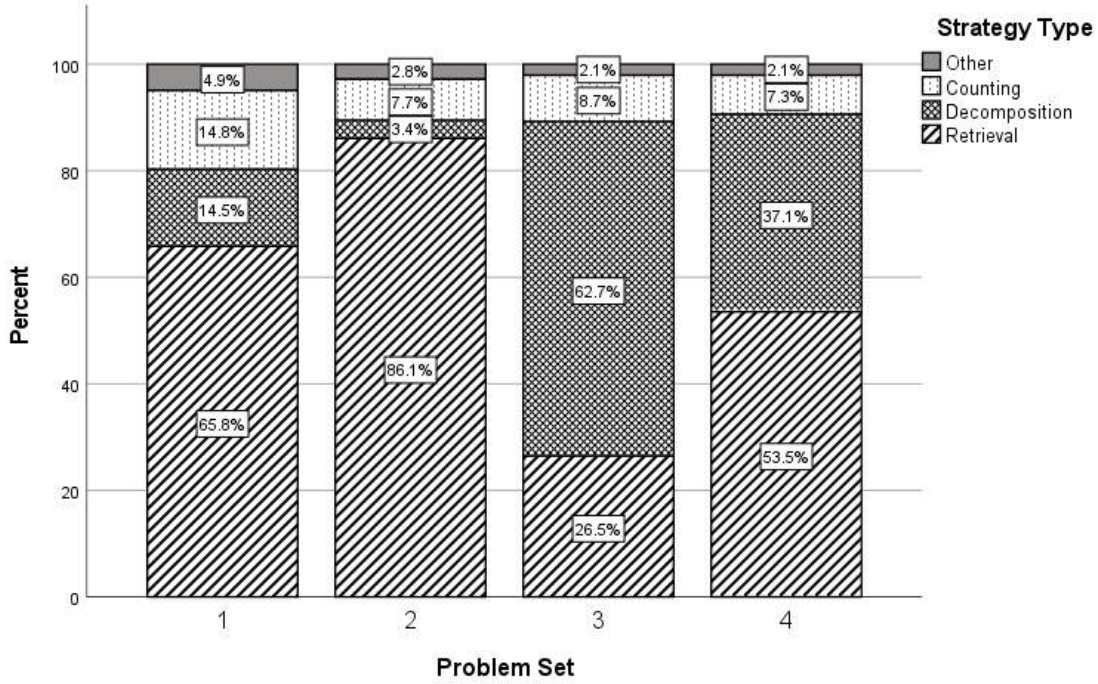


Figure 2. Addition strategies by type and problem set.

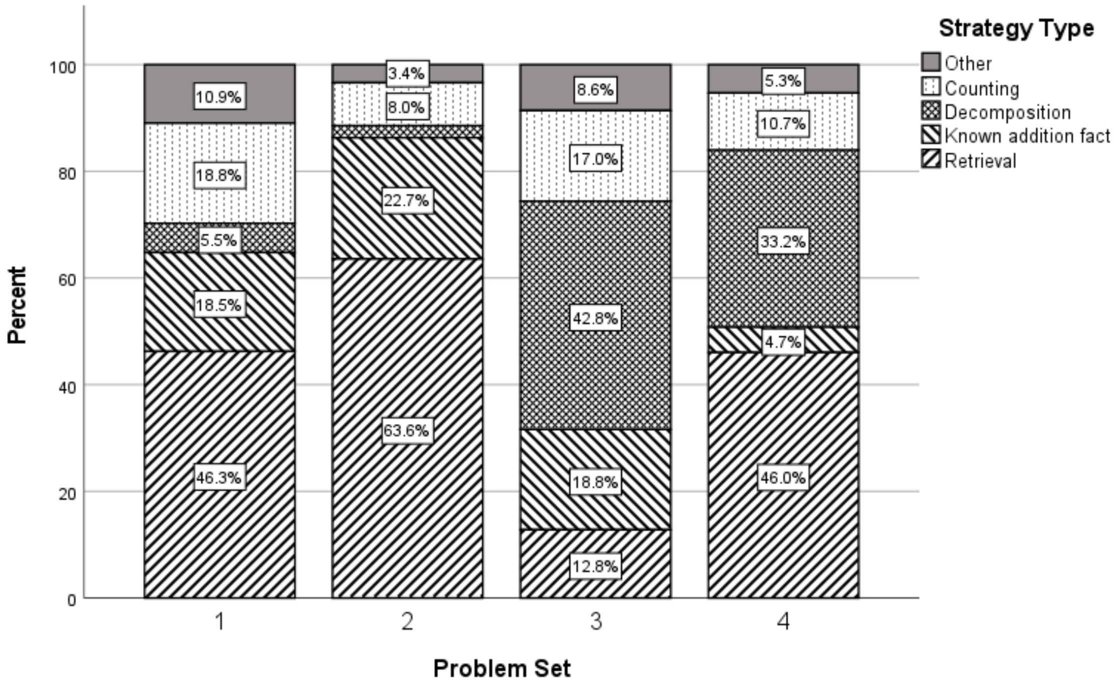


Figure 3. Subtraction strategies by type and problem set.

Students' prevalent use of Counting Strategies for addition (see Figure 2) is noteworthy given the age (year level) of participants; however, the frequency of counting recorded here is somewhat lower than that documented for similar aged students (Hopkins et al., 2022). This difference may reflect the fact that participating schools represented communities of relatively high SES. The prevalent use of Decomposition Strategies shown in Figure 2 is also noteworthy. It is surprising to see students using Decomposition Strategies at all on problems with sums less than ten (14.5% of trials). Another noteworthy finding was that participants used Decomposition Strategies most commonly to solve addition problems in Problem Set 3 (62.7% of trials). This finding highlights how it is unreasonable to expect students to recall addition facts for all single-digit numbers, as previously suggested in the Australian Curriculum (v. 8.4; ACARA, n.d.) and currently suggested in the US curriculum (CCSSO, 2010).

Comparing the frequency of students' addition strategies with their subtraction strategies reveals rates of retrieval for addition problems are similar to rates of retrieval, combined with applying a known addition fact, for corresponding subtraction across all problems sets. This finding suggests that students who solve addition problems using retrieval solve the corresponding subtraction problems using either direct retrieval or by applying the known addition fact. Comparing addition and subtraction strategies further suggests that for addition problems that are solved using Decomposition Strategies, corresponding subtraction problems are solved using Decomposition Strategies or Counting Strategies. Clearly, there are opportunities to interrogate these data further. We expect our findings will highlight the need for greater clarity around expectations to ensure students learn to retrieve the most important facts and develop efficient Decomposition Strategies for addition and subtraction within 20.

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